The Inductance of a Coaxial Transmission Line

Recall that we earlier determined the **capacitance** (per unit length) of a **coaxial transmission line** to be:

$$\frac{C}{\ell} = \frac{2\pi\varepsilon}{\ln[b/a]} \qquad \left[\frac{\text{farads}}{\text{meter}}\right]$$

We can likewise determine its **inductance** per unit length.

Q: *Yikes! How do we accomplish this? There are no* ***loops*** *in a coaxial line!*

A: True. We instead begin by determining the **energy stored** (per unit length) of a coax line.

Recall that the magnetic flux density **between** the inner and outer conductors of a coaxial line is:

$$\mathbf{B}(\bar{r}) = \frac{\mu I}{2\pi\rho}\,\hat{a}_\phi \qquad (a < \rho < b)$$

Therefore the **magnetic field** within the line is:

$$\mathbf{H}(\bar{r}) = \frac{I}{2\pi\rho}\,\hat{a}_\phi \qquad (a < \rho < b)$$

The **energy stored** in a length ℓ of the coax line is therefore:

$$
\begin{aligned}
W_m &= \frac{1}{2}\iiint \mathbf{B}\cdot\mathbf{H}\, dv \\
&= \frac{\mu I^2}{8\pi^2}\int_0^{\ell}\int_a^{b}\int_0^{2\pi} \frac{1}{\rho^2}\,\hat{a}_\phi\cdot\hat{a}_\phi\,\rho\, d\rho\, d\phi\, dz \\
&= \frac{\mu I^2 \ell}{4\pi}\ln\left[\frac{b}{a}\right]
\end{aligned}
$$

Q: *So what? We want to find the* ***inductance*** *of the line,* ***not*** *the energy stored in it!*

A: True. But recall inductance is **related** to stored energy as:

$$W_m = \frac{1}{2} L I^2$$

Or in other words:

$$L = \frac{2W_m}{I^2}$$

Using this expression, we find:

$$
\begin{aligned}
L &= \frac{2}{I^2}\left(\frac{\mu I^2 \ell}{4\pi}\ln\left[\frac{b}{a}\right]\right) \\
&= \frac{\mu}{2\pi}\ln\left[\frac{b}{a}\right]\ell
\end{aligned}
$$

Or, in other words, the **inductance per unit length** of a coax transmission line is:

$$\frac{L}{\ell} = \frac{\mu}{2\pi} \ln\left[\frac{b}{a}\right] \qquad \left[\frac{\text{Henries}}{\text{m}}\right]$$

Note here that we did **not** consider the magnetic fields **within the conductors**. For most engineering applications (i.e., **time-varying**), we will find that the contribution of these fields are small and thus can be **neglected**.